BEST

Current Electricity

S 8 : Combination of Cells

What you already know

- Charging and discharging of cell
- Examples on cells
- Series combination of cells - Part 1

What you will learn

- Series combination of cell - Part 2
- Parallel combination of cells - Part 1

Recap

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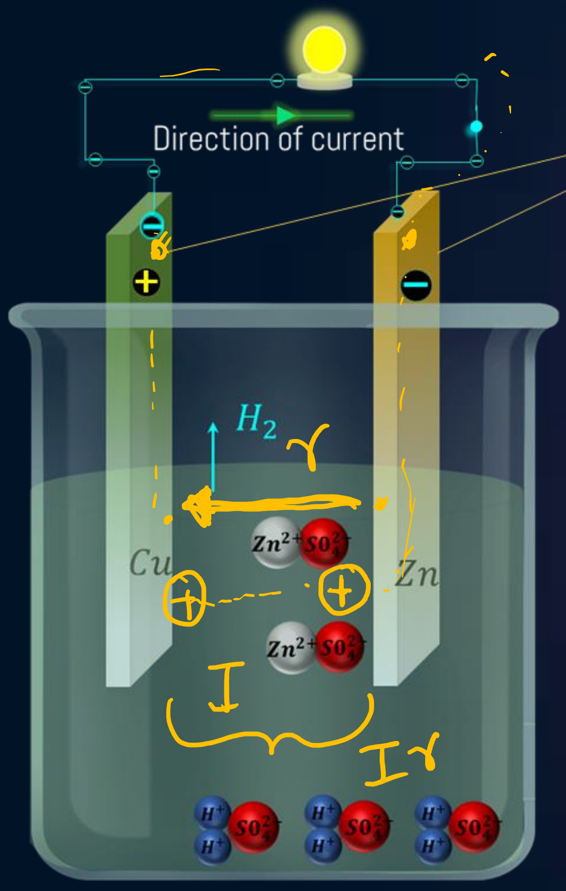
Emf of a cell



$$\mathcal{E} = V + Ir$$

EMF of a cell is defined as work done by cell in moving unit positive charge in the whole circuit including the cell once.

$$E = \frac{W}{q}$$

S.I Unit $= \dfrac{Joule}{Coulomb}$ or $Volt$

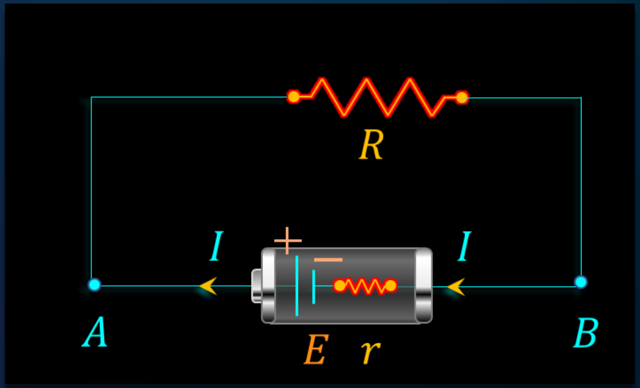
$$\boxed{V = \mathcal{E} - Ir}$$

Emf is independent of quantity of electrolyte, size of electrodes and distance between the electrodes.

Recap

Relation b/w Emf and voltage

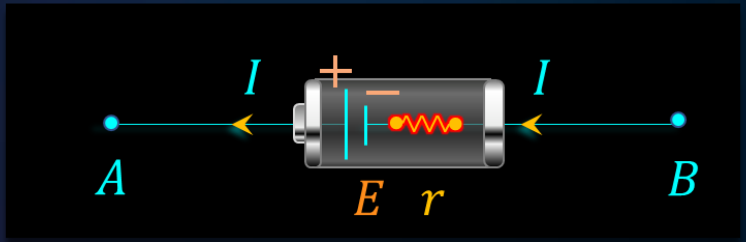
$$\boxed{V = E - Ir}$$



Discharging of a cell

$$I = \frac{E}{R + r}$$

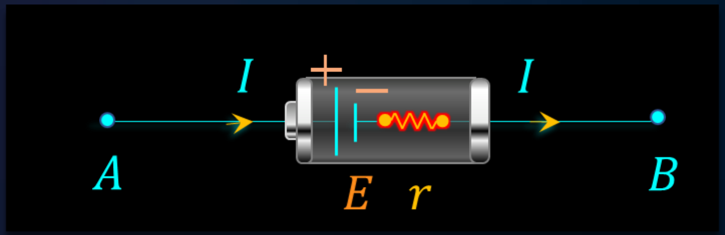
$$V < E$$

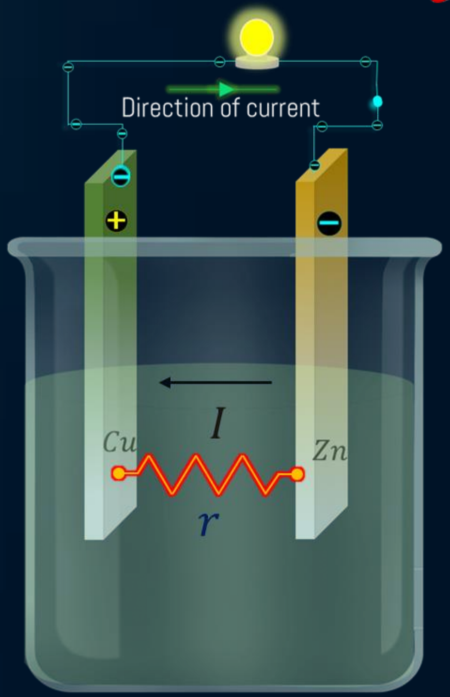


Charging of a cell

$$V > E$$

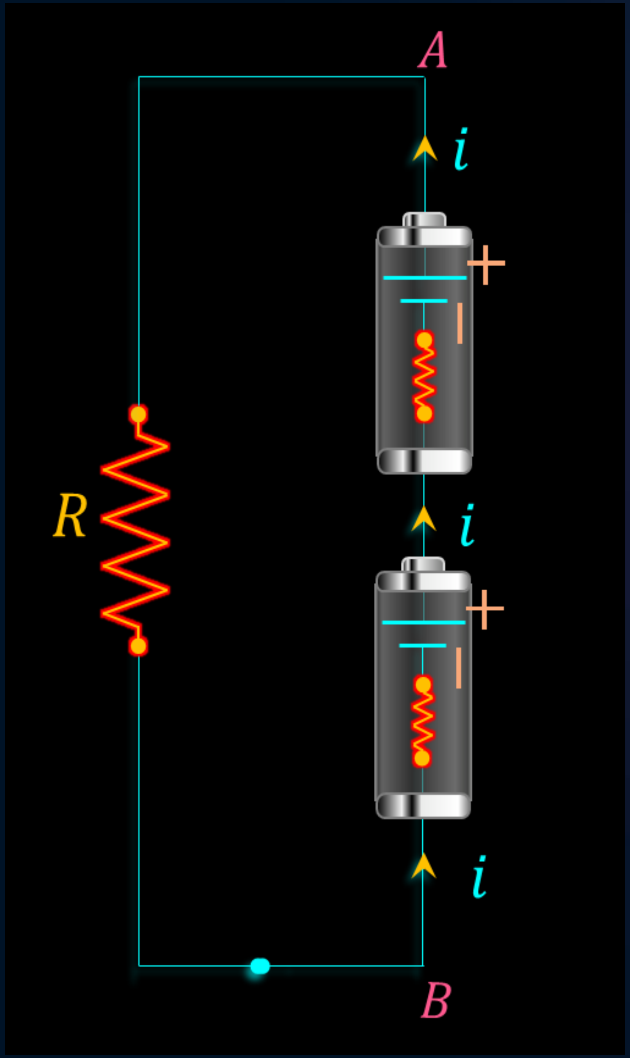
$$\boxed{V = E + Ir}$$





Recap

Series combination of cells



Same polarity:

Same current flow through all cells

$$\boxed{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2} \qquad \boxed{r_{eq} = r_1 + r_2}$$

If n non-identical cells are connected in series with same polarity,

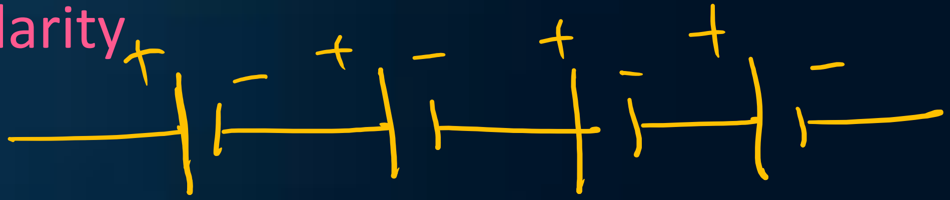
$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \cdots \varepsilon_n$$

$$r_{eq} = r_1 + r_2 + r_3 + \cdots r_n$$

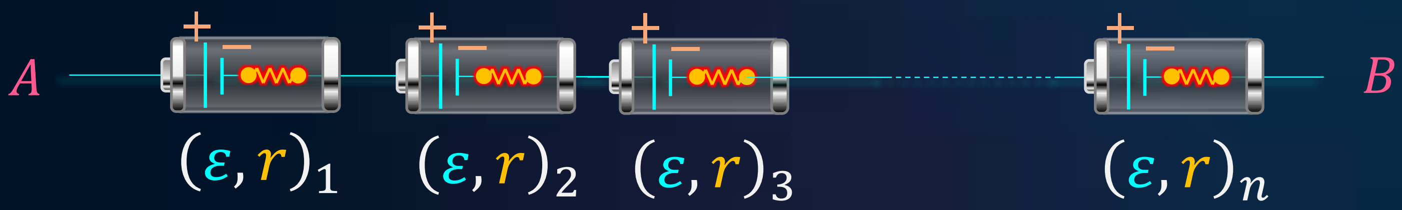
$$\boxed{\varepsilon_{eq} = \sum_{i=1}^{n} \varepsilon_i} \qquad \boxed{r_{eq} = \sum_{i=1}^{n} r_i}$$

Series Combination of Cells

- n identical cells are connected in series with same polarity

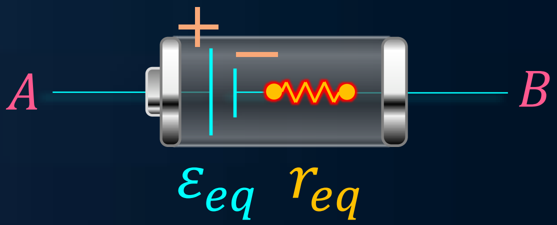


Series



$$\varepsilon_{eq} = n\varepsilon$$

$$r_{eq} = nr$$



$$\boxed{\varepsilon_{eq} = n\varepsilon}$$

$$\boxed{r_{eq} = nr}$$

Series Combination of Cells

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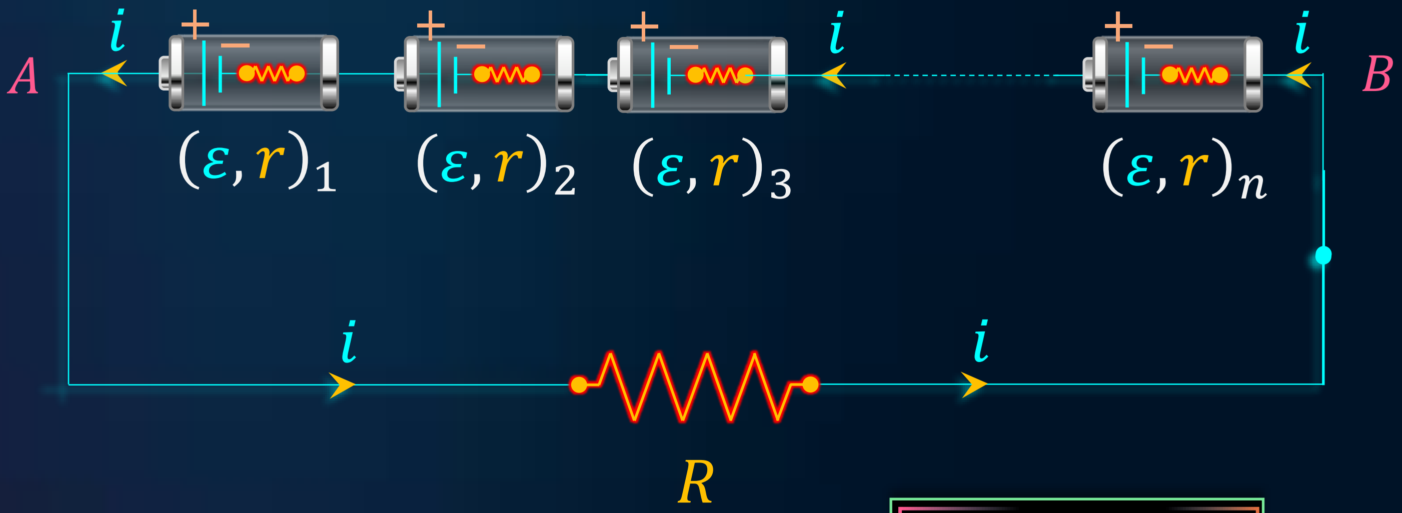
- n identical cells are connected in series with same polarity

$$\boxed{\varepsilon_{eq} = n\varepsilon} \qquad \boxed{r_{eq} = nr}$$



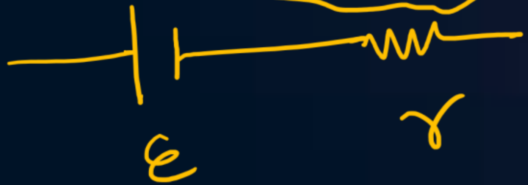
$$R_{eq} = R + nr$$

$$\boxed{i = \frac{\varepsilon_{eq}}{R_{eq}} = \frac{n\varepsilon}{R + nr}}$$

$$\boxed{i = \frac{n\varepsilon}{R + nr}}$$

$$R >> nr \longrightarrow i = \frac{n\varepsilon}{R}$$

$$i = \frac{\varepsilon}{r}$$



$$nr >> R \longrightarrow i = \frac{\cancel{n}\varepsilon}{\cancel{n}r} = \frac{\varepsilon}{r}$$

Series Combination of Cells

- n identical cells are connected in series with same polarity

$$\boxed{i = \frac{\varepsilon}{r}}$$

No change in current than current in single cell

Operating voltage

$12\ V$

$nr \gg R$

$1.2\ V \rightarrow 12\ V$

$1\ A$

Example:

$$\varepsilon = 1.2\ V$$

$$n = 10$$

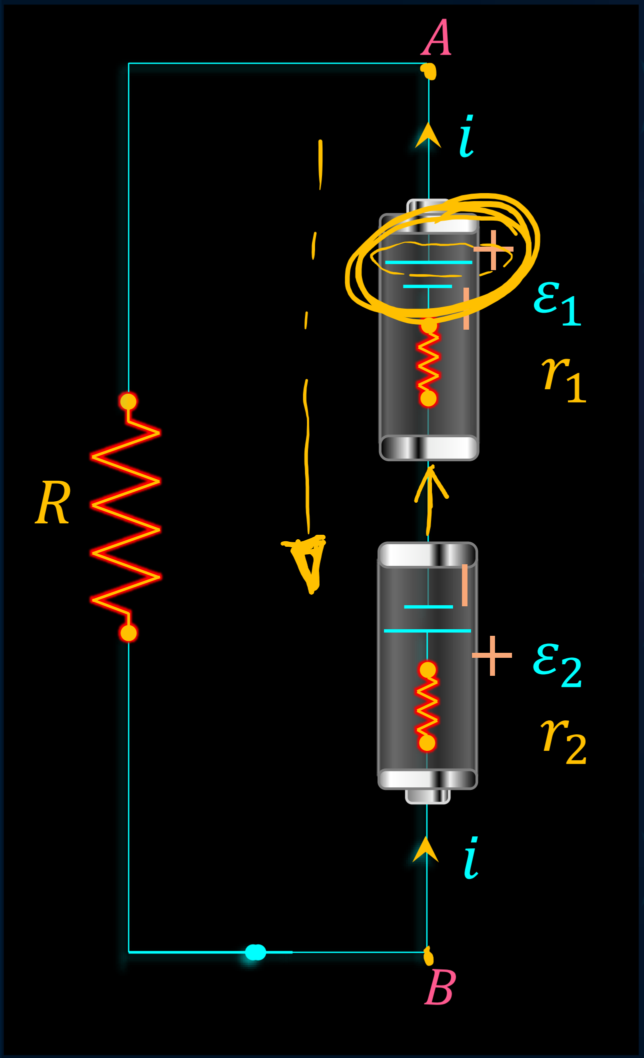
$$\varepsilon_{eq} = 10 \times 1.2 = 12\ V$$

$n\varepsilon = 12$

$n \cdot (1.2) = 12$

$n = 10$

Series Combination of Cells



- Opposite polarity
- Same current flow through all cells

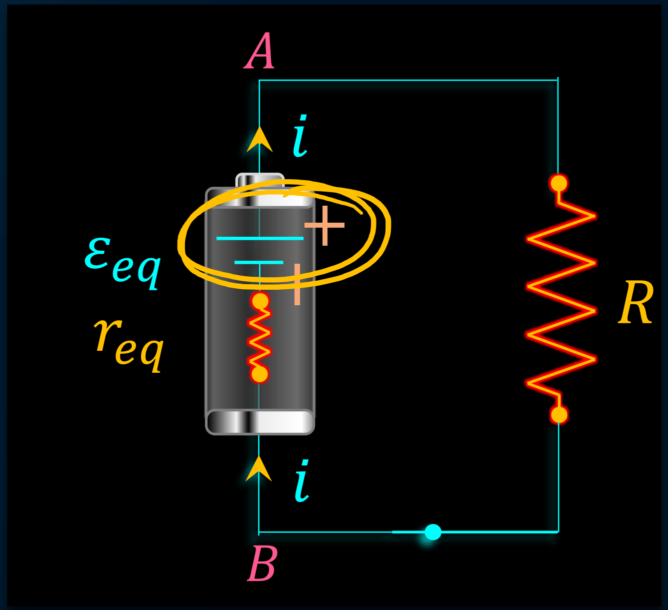
$$V_A - \varepsilon_1 + i r_1 + \varepsilon_2 + i r_2 = V_B$$

$$V_A - V_B = \varepsilon_1 - \varepsilon_2 - i(r_1 + r_2)$$

- $\boxed{\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2}$

- $\boxed{r_{eq} = r_1 + r_2}$

$$V = \varepsilon_{eq} - i r_{eq}$$

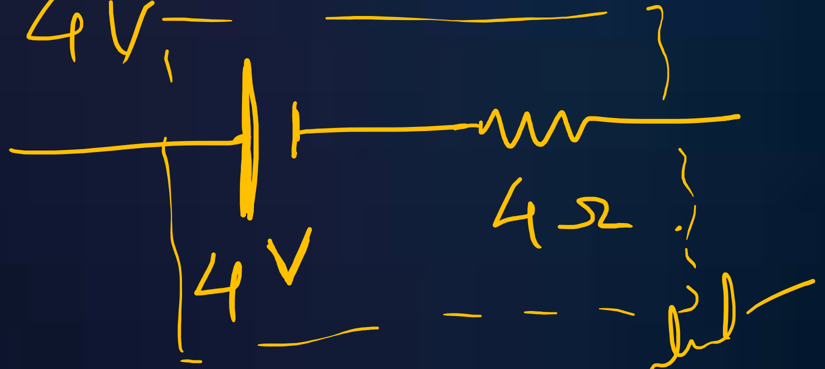


Series Combination of Cells

$r_{eq} = 4\Omega$

$\varepsilon_{eq} = +6V - 4V + 2V$

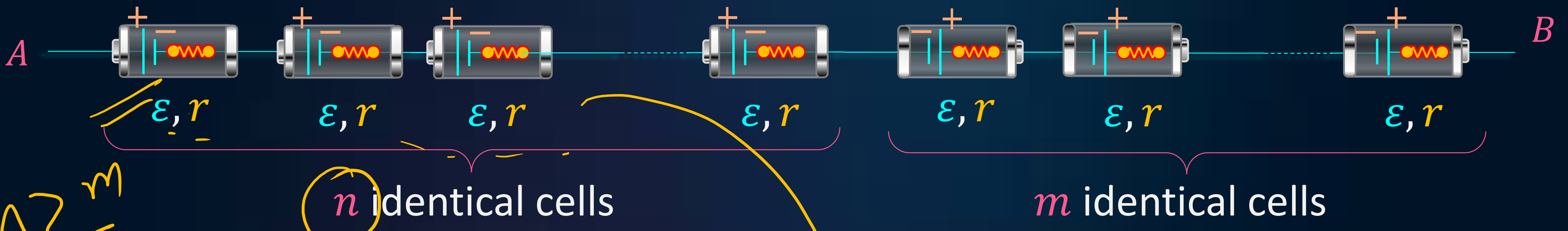
$= 4V$



Equivalent

Series Combination of Cells

- n identical cells with same polarity and m same cells with opposite polarity

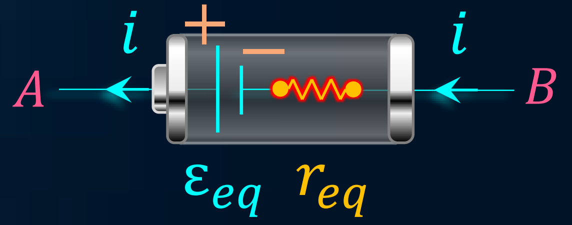


$n > m$

$$\varepsilon_{eq} = n\varepsilon - m\varepsilon$$

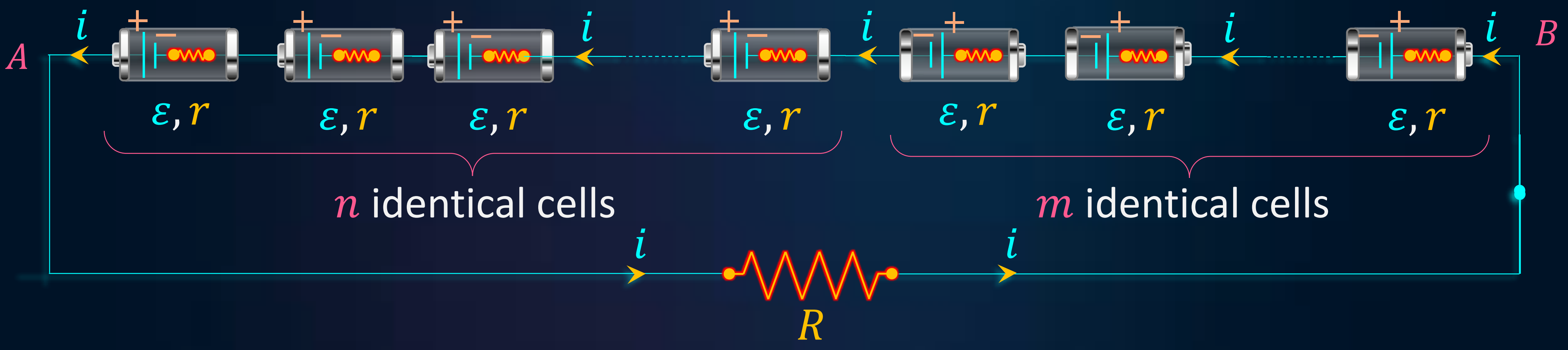
$$= (n - m)\varepsilon$$

$$r_{eq} = (n + m)r$$



Series Combination of Cells

- n identical cells with same polarity and m same cells with opposite polarity



$$\boxed{\varepsilon_{eq} = n\varepsilon - m\varepsilon}$$

$$\boxed{r_{eq} = (n+m)r}$$

$$R_{eq} = R + r_{eq}$$

$$\boxed{R_{eq} = R + (n+m)r}$$

$$i = \frac{\mathcal{E}_{eq}}{R_{eq}} = \frac{(n-m)\,\varepsilon}{R+(n+m)r}$$

Question

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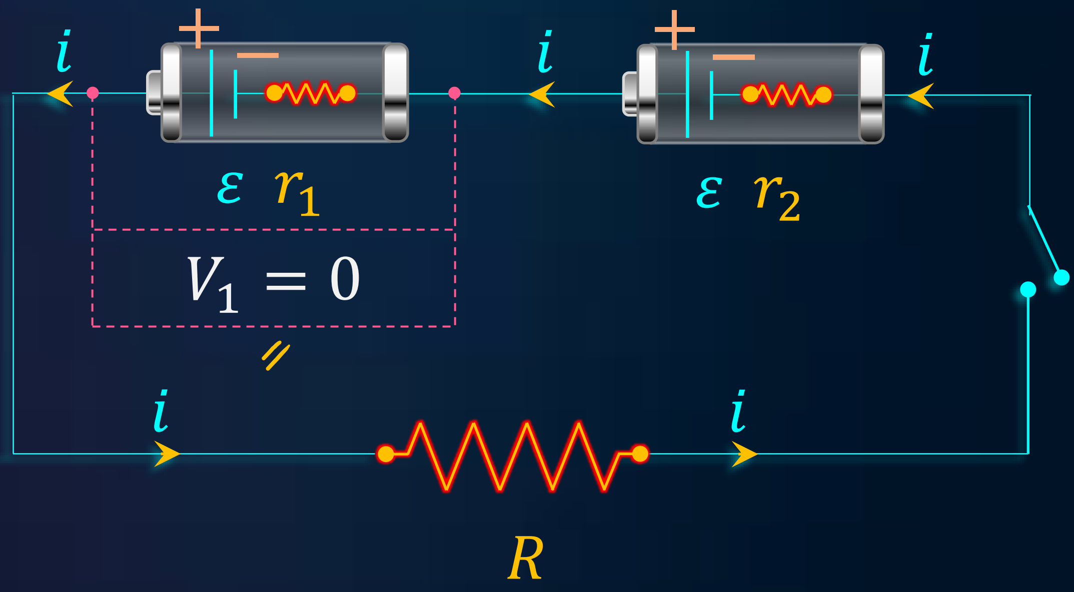
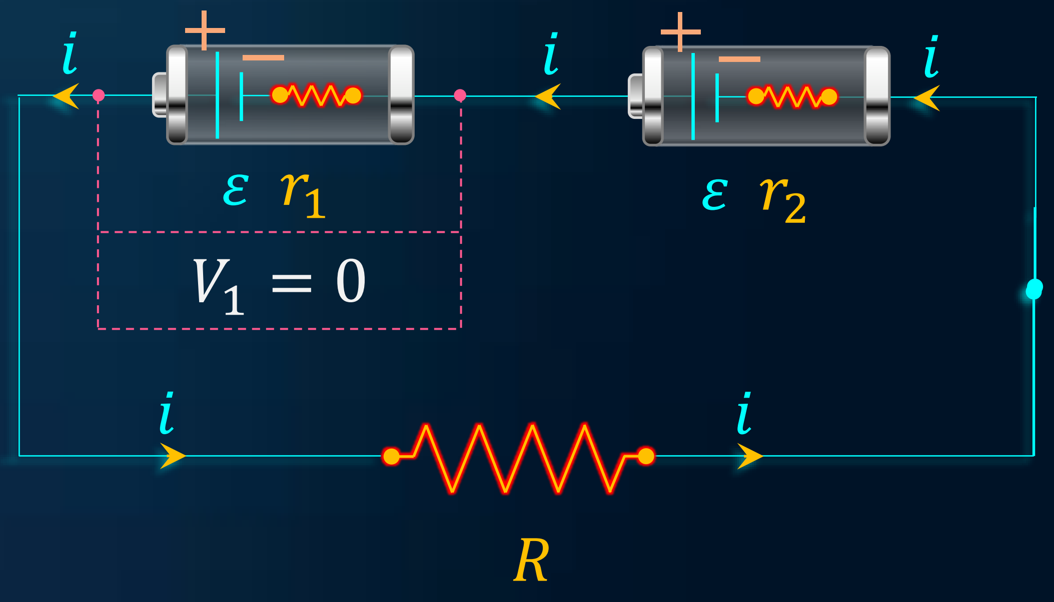
Two cells, having the same e.m.f. are connected in series through an external resistance R. Cells having internal resistances r_1 and r_2 $(r_1 > r_2)$ respectively. When the circuit is closed, the potential difference across the first cell is zero. The value of R is:

- a) $r_1 + r_2$
- b) $r_1 - r_2$
- c) $\frac{r_1 + r_2}{2}$
- d) $\frac{r_1 - r_2}{2}$



Discussion

External resistance and cell resistances are in series connection



$$R_{eq} = R + r_1 + r_2$$

$$\cancel{V_1}^{\,0} = \varepsilon - i r_1$$

$$i = \frac{\varepsilon_{eq}}{R_{eq}} = \frac{2\varepsilon}{R + r_1 + r_2}$$

$$\cancel{V_1}^{\,0} = \varepsilon - \frac{2\varepsilon r_1}{R + r_1 + r_2}$$

Discussion

$$i = \frac{2\varepsilon}{R + r_1 + r_2}$$

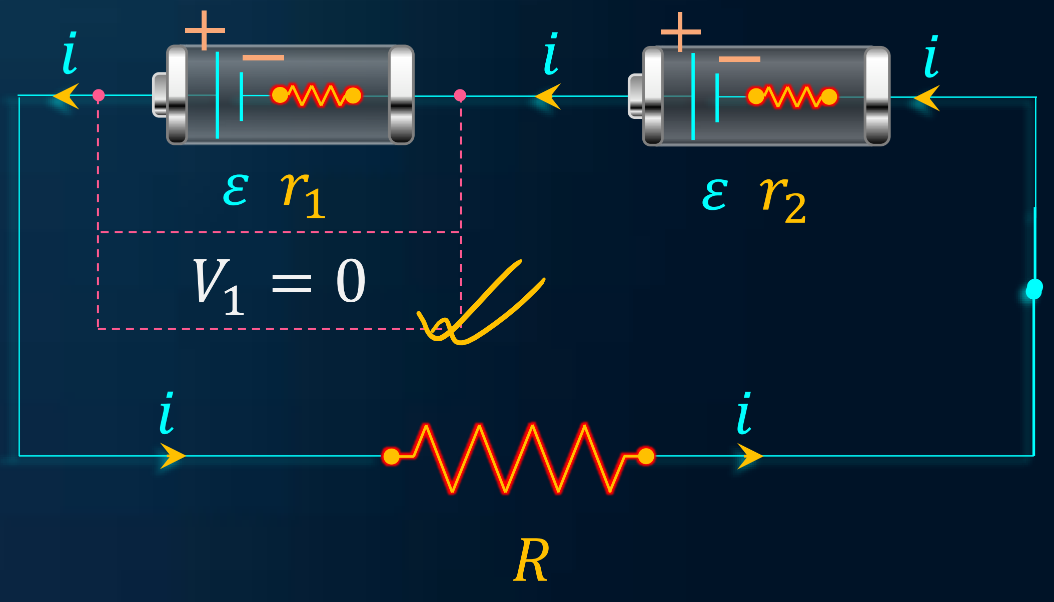
Terminal p.d. across first cell is:

$$V_1 = \varepsilon - i r_1$$

$$\cancel{\varepsilon} = \frac{2\cancel{\varepsilon} r_1}{R + r_1 + r_2}$$

$$R + r_1 + r_2 = 2r_1$$

$$\boxed{R = r_1 - r_2}$$



Thus, option (b) is the correct answer.

Question

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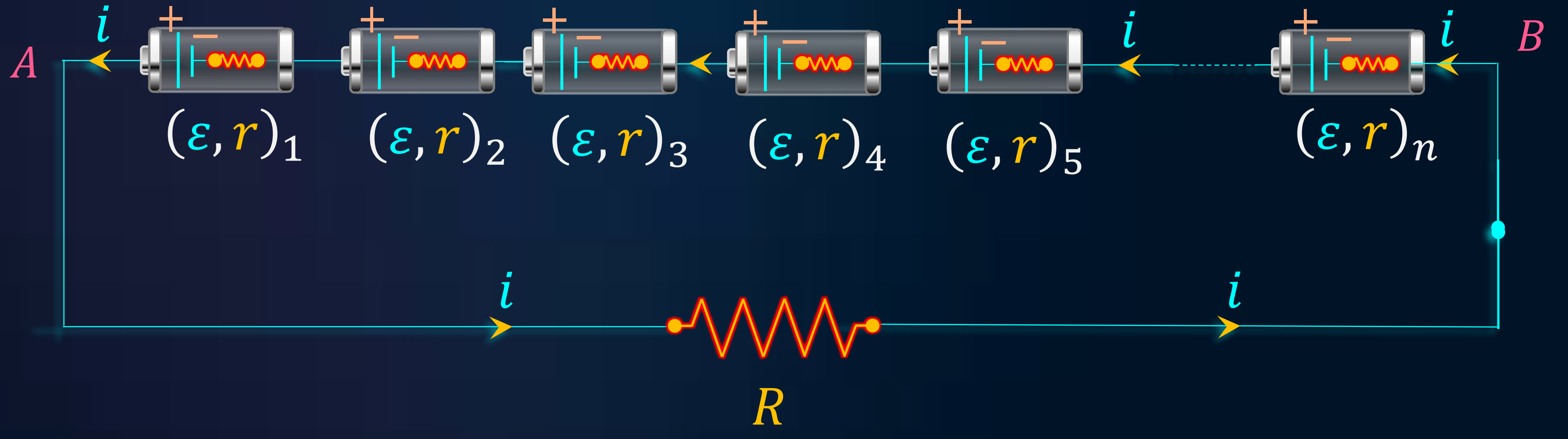
$'n'$ cells each of e.m.f $'\varepsilon'$ and internal resistance $'r'$ are connected in series in same polarity. An external resistance R is connected to the combination. If the polarity of $'m'$ cells are reversed, find current in the external resistor.

a) $\dfrac{(n-m)\varepsilon}{R+(n+m)r}$

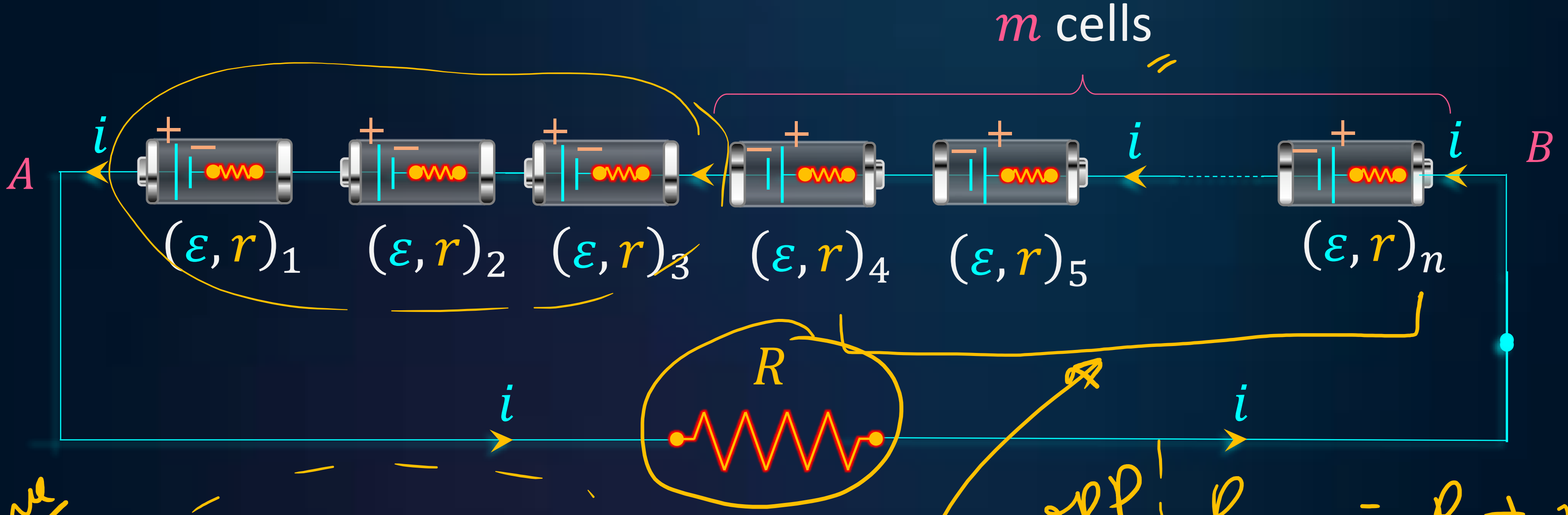
b) $\dfrac{n\,\varepsilon}{R+nr}$

c) $\dfrac{(n-m)\varepsilon}{R+nr}$

d) $\dfrac{(n-2m)\varepsilon}{R+nr}$



Discussion



Same

$+(n-m)\varepsilon$

$m\varepsilon$ opp

$R_{eq} = R + nr$

$$E_{eq} = (n-m)\varepsilon - m\varepsilon$$
$$= n\varepsilon - m\varepsilon - m\varepsilon$$
$$= (n-2m)\varepsilon$$

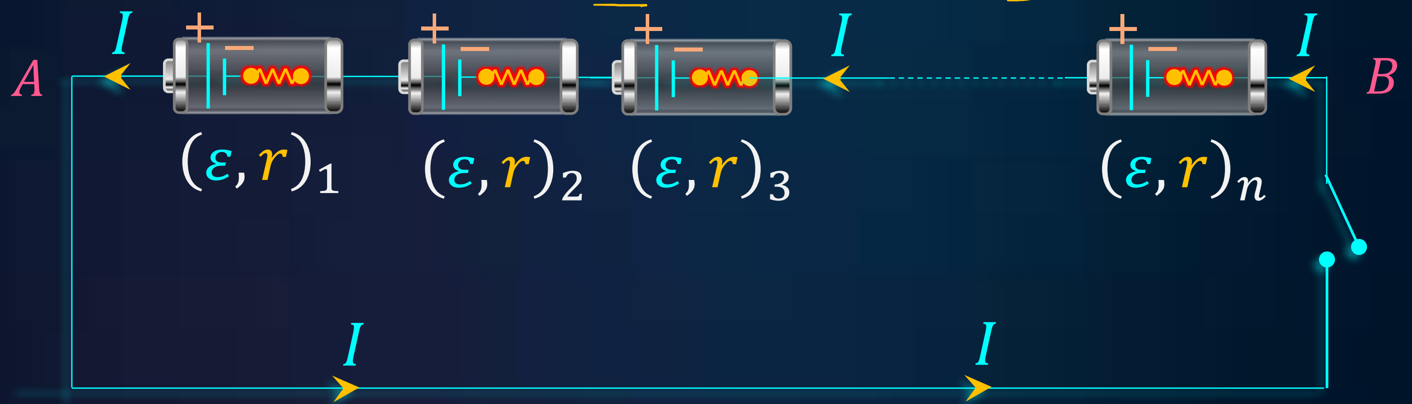
$$i = \frac{E_{eq}}{R_{eq}}$$

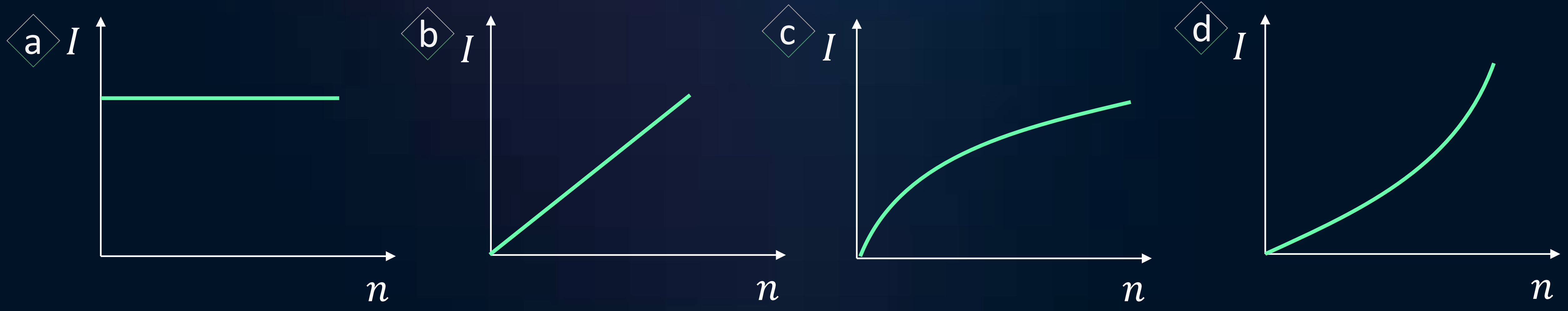
$$i = \frac{(n-2m)\varepsilon}{R+nr}$$

Thus, option (d) is the correct answer.

Question

A battery consists of a variable $'n'$ number of identical cells (having internal resistance $'r'$ each) which are connected in series. The terminals of battery are short circuited and the current I is measured. Which of the graphs shows the correct relationship between I and n.





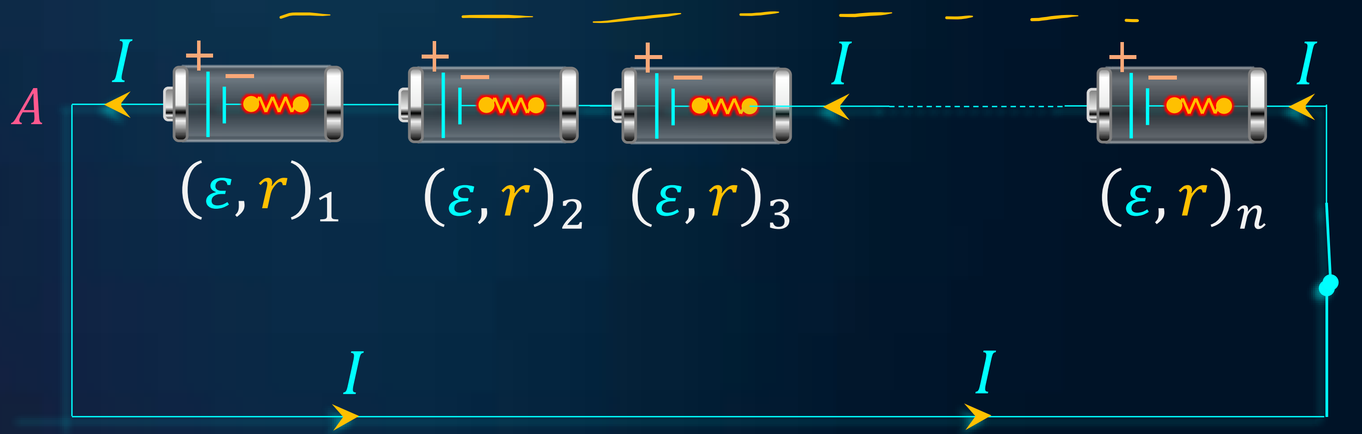
Discussion

$\varepsilon_{eq} = n\varepsilon$

$r_{eq} = nr$

$R_{eq} = \overset{0}{\cancel{R}} + r_{eq}$

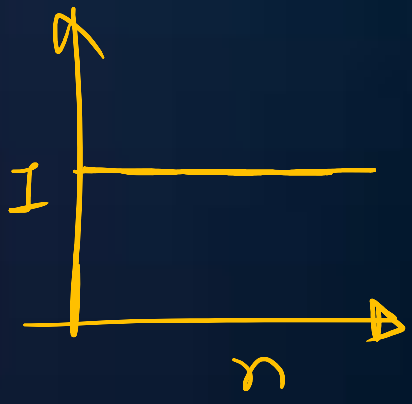
$= r_{eq}$



$\boxed{i = \frac{\varepsilon}{r}}$

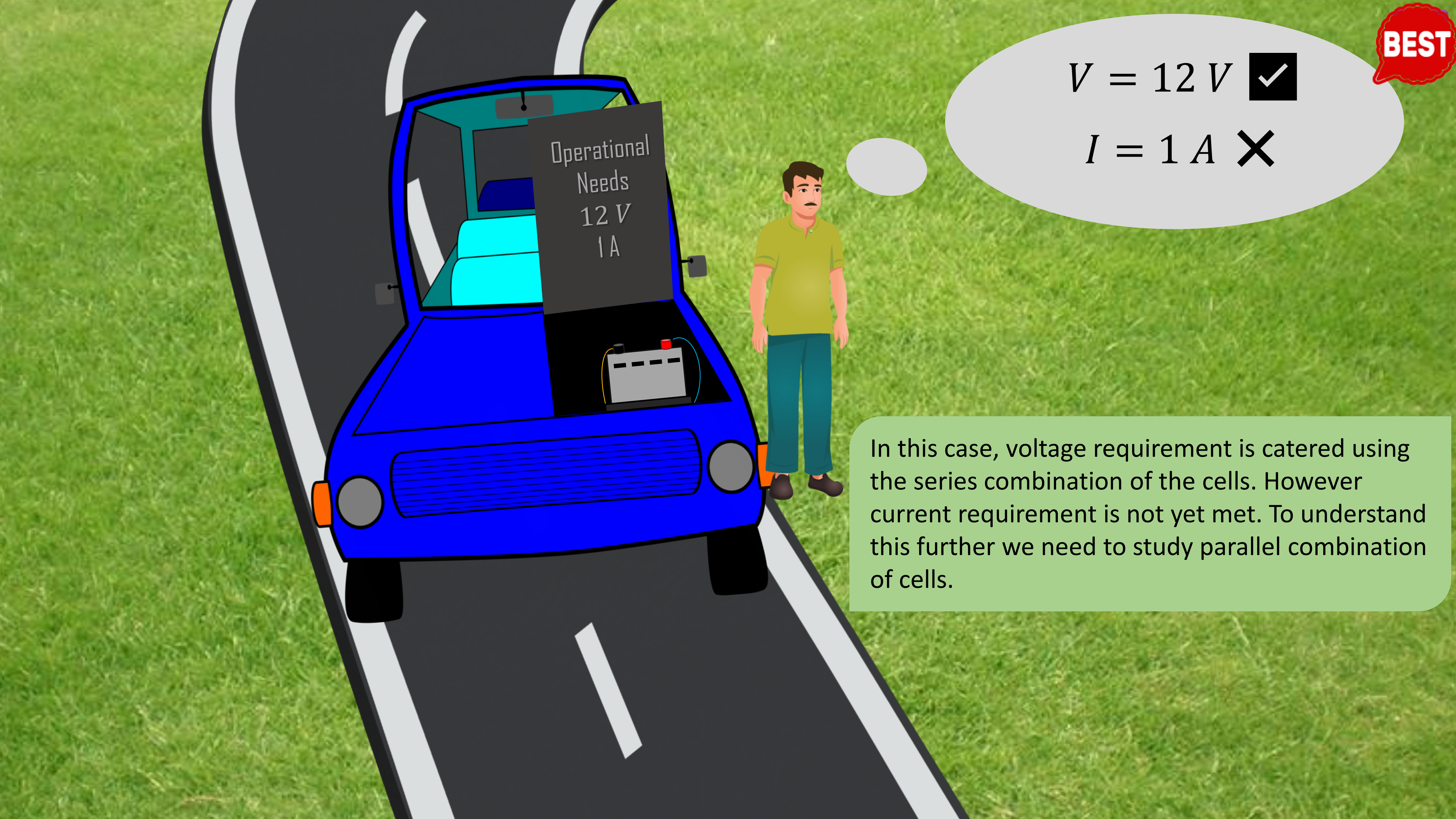
$i = \frac{\varepsilon}{r}$

$i = \frac{\varepsilon_{eq}}{R_{eq}} = \frac{\cancel{n}\varepsilon}{\cancel{n}r}$



Thus, option (a) is the correct answer.

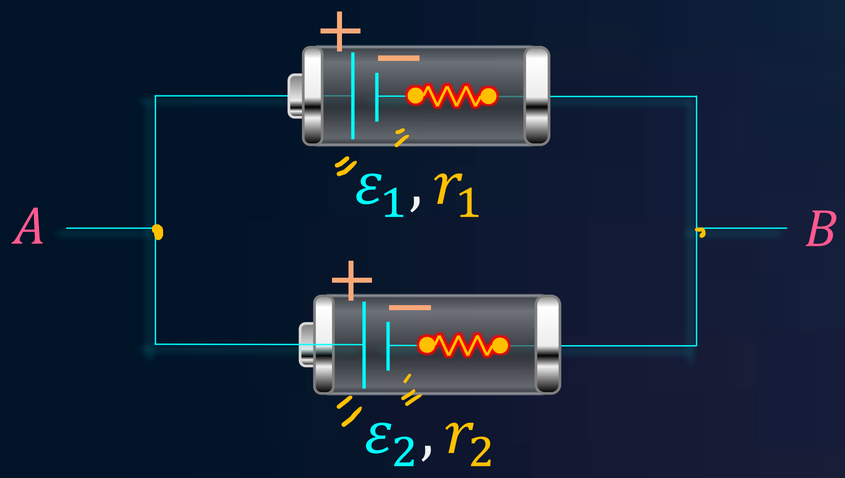
∴ Current I is constant and independent of n



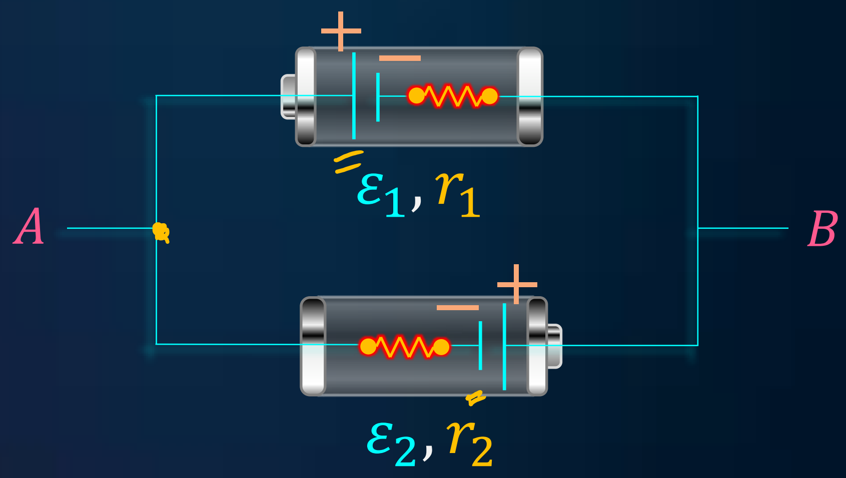
In this case, voltage requirement is catered using the series combination of the cells. However current requirement is not yet met. To understand this further we need to study parallel combination of cells.

Parallel Combination of Cells

Polarity



Same polarity



Opposite polarity

Parallel Combination of Cells

$i = i_1 + i_2$ ....(1)

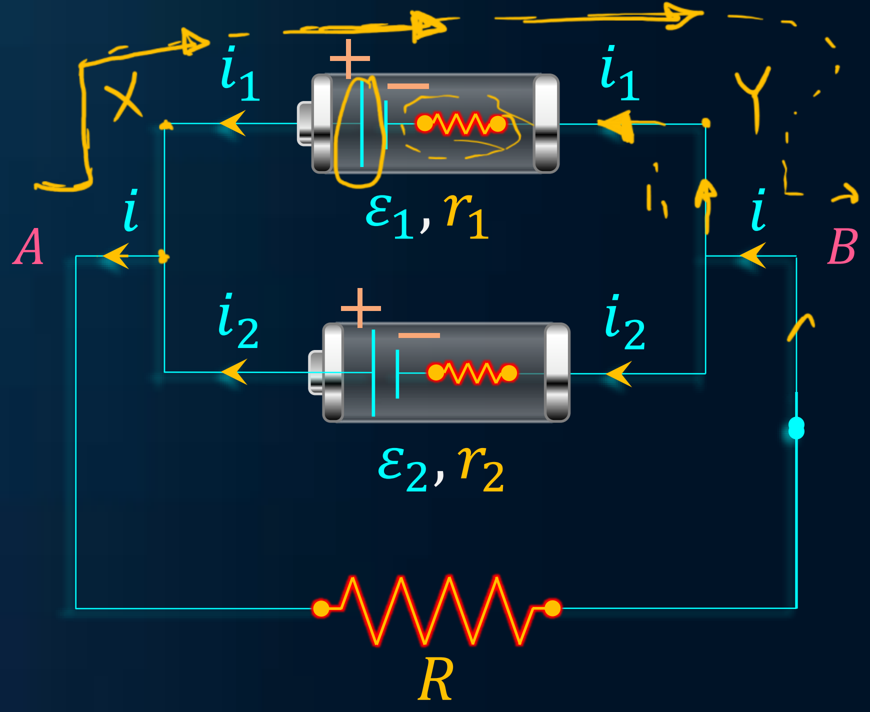
Apply KVL in path A X Y B

$$V_A - \varepsilon_1 + i_1 r_1 = V_B$$

$$V_A - V_B = \varepsilon_1 - i_1 r_1$$

$$V = \varepsilon_1 - i_1 r_1$$

$$i_1 = \frac{\varepsilon_1 - V}{r_1} \quad \text{—— (2)}$$



Same polarity cells

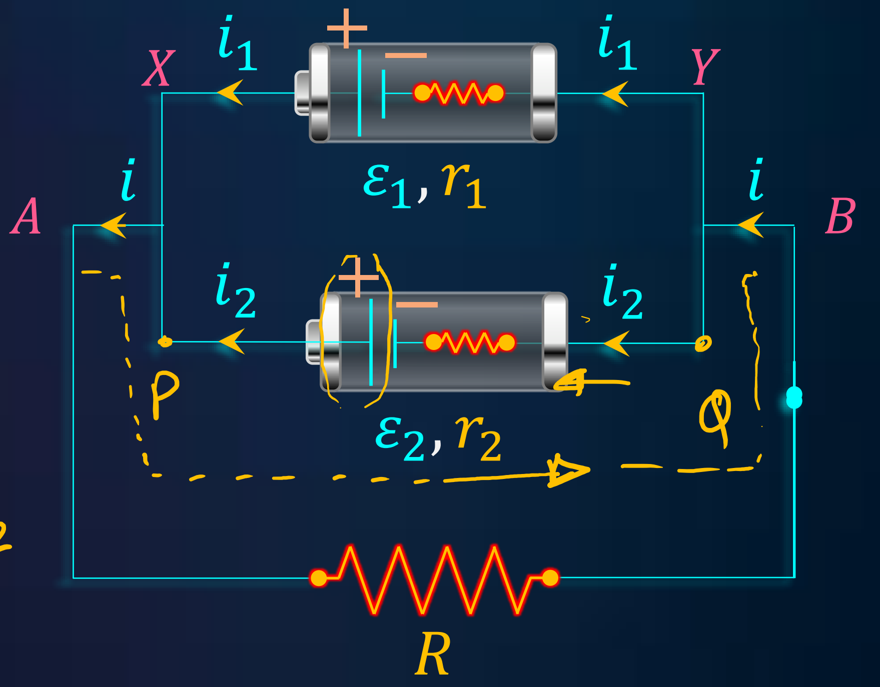
Parallel Combination of Cells

$$i = i_1 + i_2 \quad \text{.........}(1)$$

$$i_1 = \frac{\varepsilon_1 - V}{r_1} \quad \text{.........}(2)$$

APQB

$$V = \varepsilon_{eq} - I r_{eq}$$



$$V_A - \varepsilon_2 + i_2 r_2 = V_B$$

$$V_A - V_B = \varepsilon_2 - i_2 r_2$$

$$V = \varepsilon_2 - i_2 r_2$$

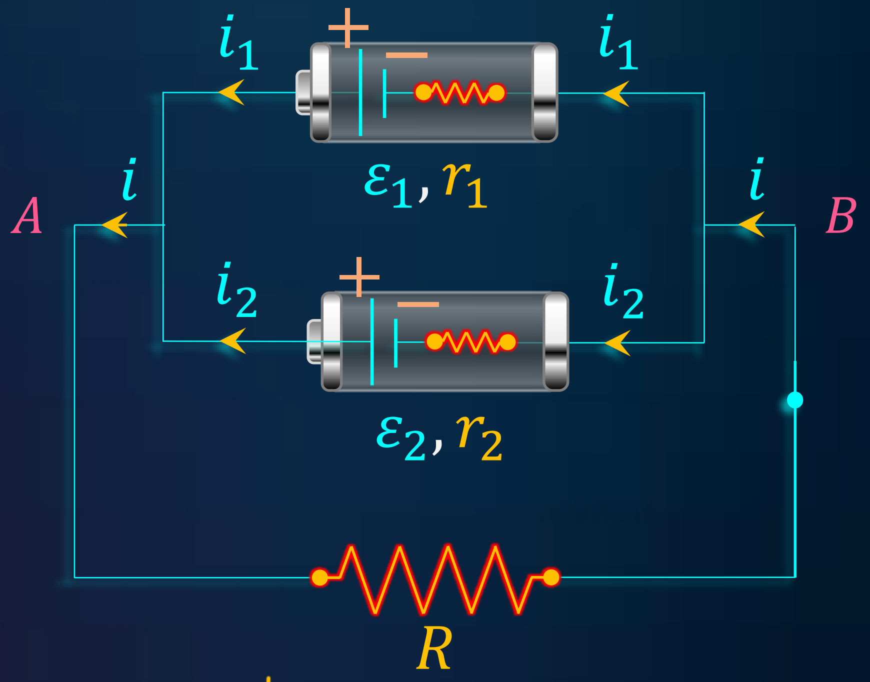
$$i_2 = \frac{\varepsilon_2 - V}{r_2} \quad — (3)$$

Parallel Combination of Cells

$$i = i_1 + i_2 \quad \text{.........(1)}$$

$$i_1 = \frac{\varepsilon_1 - V}{r_1} \quad \text{.........(2)}$$

$$i_2 = \frac{\varepsilon_2 - V}{r_2} \quad \text{.........(3)}$$



$$V = \varepsilon - Ir$$

$$i = \frac{\varepsilon_1}{r_1} - \frac{V}{r_1} + \frac{\varepsilon_2}{r_2} - \frac{V}{r_2}$$

$$i = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

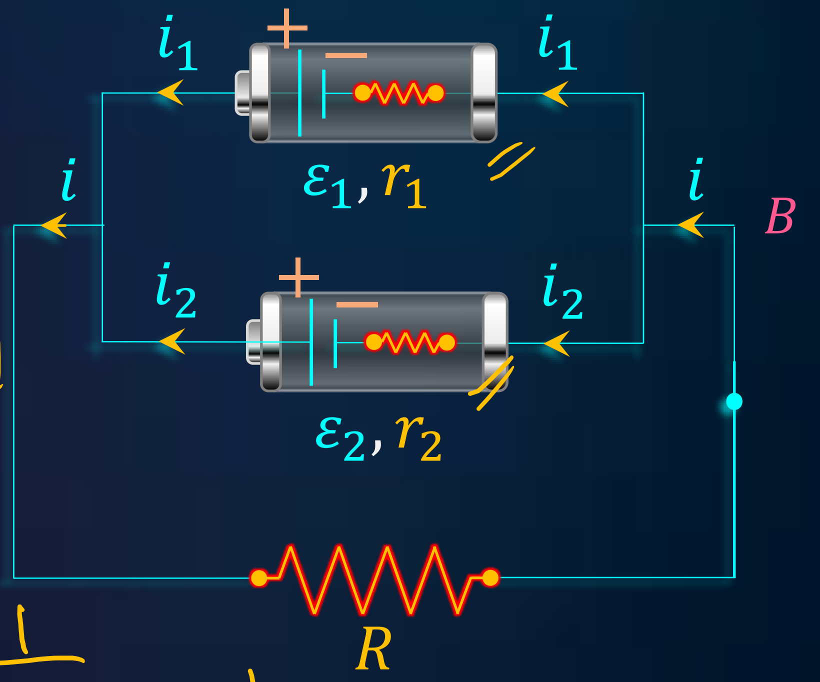
$$V\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - i$$

Parallel Combination of Cells

$$V\left(\frac{1}{r_1}+\frac{1}{r_2}\right)=\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}-i$$

$$V = \varepsilon_{eq} - I r_{eq}$$

$$V = \frac{\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}} - i\left(\frac{1}{\frac{1}{r_1}+\frac{1}{r_2}}\right)$$



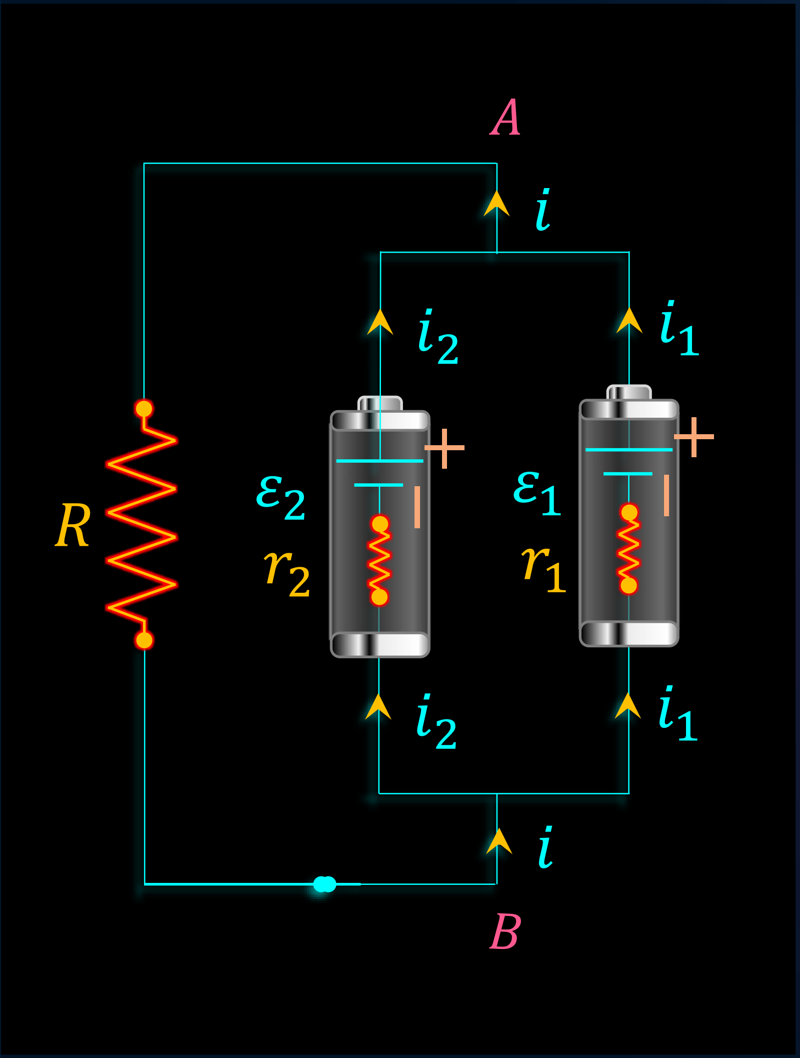
$$\boxed{\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}}}$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1}+\frac{1}{r_2}$$

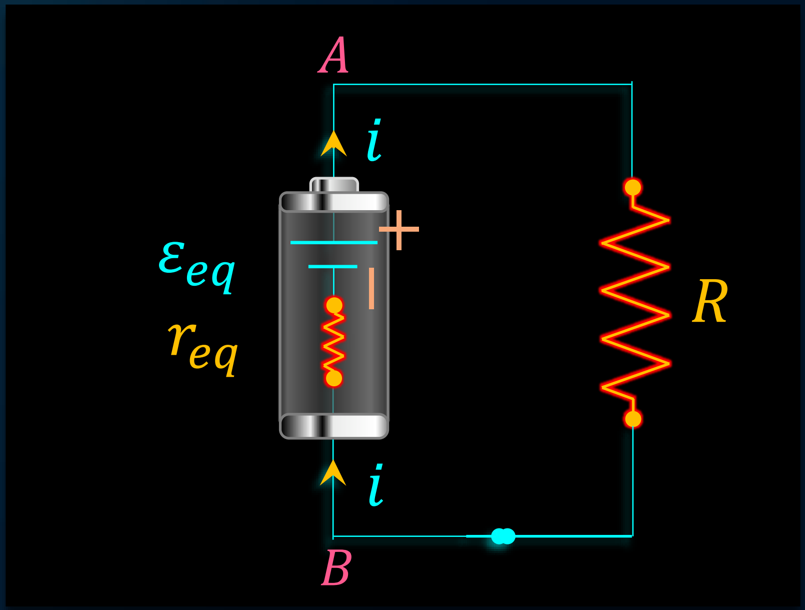
$$\boxed{\varepsilon_{eq}=\frac{\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}}}$$

$$\boxed{\frac{1}{r_{eq}}=\frac{1}{r_1}+\frac{1}{r_2}}$$

Parallel Combination of Cells

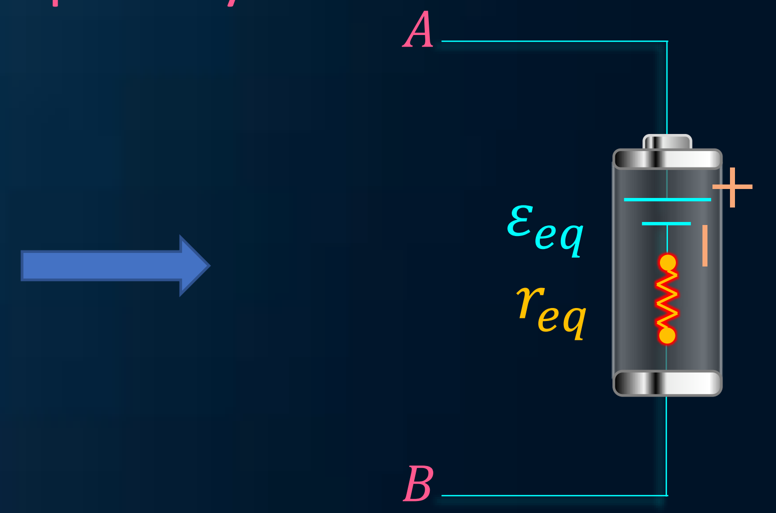


- Same polarity
- $i = i_1 + i_2$
- $$\boxed{\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}}$$
- $$\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}}$$



Parallel Combination of Cells

- n non-identical cells are connected in parallel with same polarity



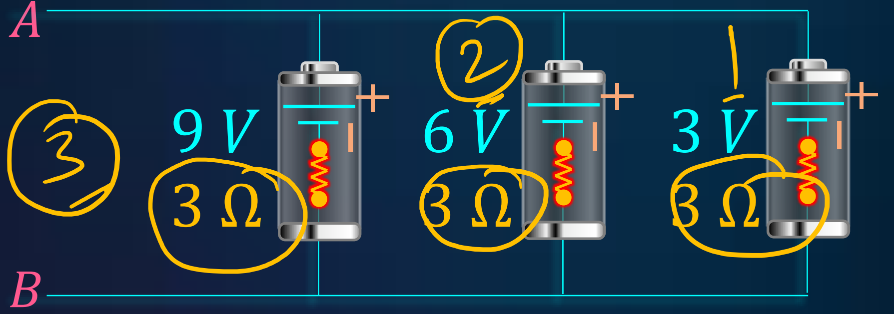
$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3} + \cdots \frac{\varepsilon_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots \frac{1}{r_n}}$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots \frac{1}{r_n}$$

$$\boxed{\varepsilon_{eq} = \frac{\sum_{i=1}^{n} \frac{\varepsilon_i}{r_i}}{\sum_{i=1}^{n} \frac{1}{r_i}}}$$

$$\boxed{\frac{1}{r_{eq}} = \sum_{i=1}^{n} \frac{1}{r_i}}$$

Parallel Combination of Cells



$$\boxed{\begin{aligned}\varepsilon_{eq} &= 6V \\ r_{eq} &= 1\Omega\end{aligned}}$$

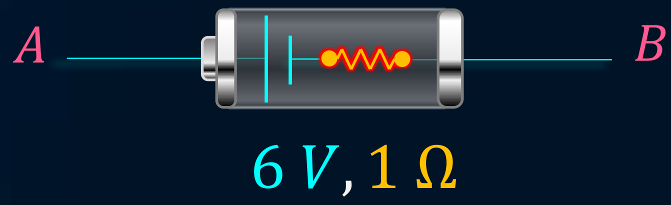
$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} \rightarrow 1$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$\frac{r}{n}$$

$$\frac{3}{3} = 1\Omega$$

$$r_{eq} = 1\Omega$$



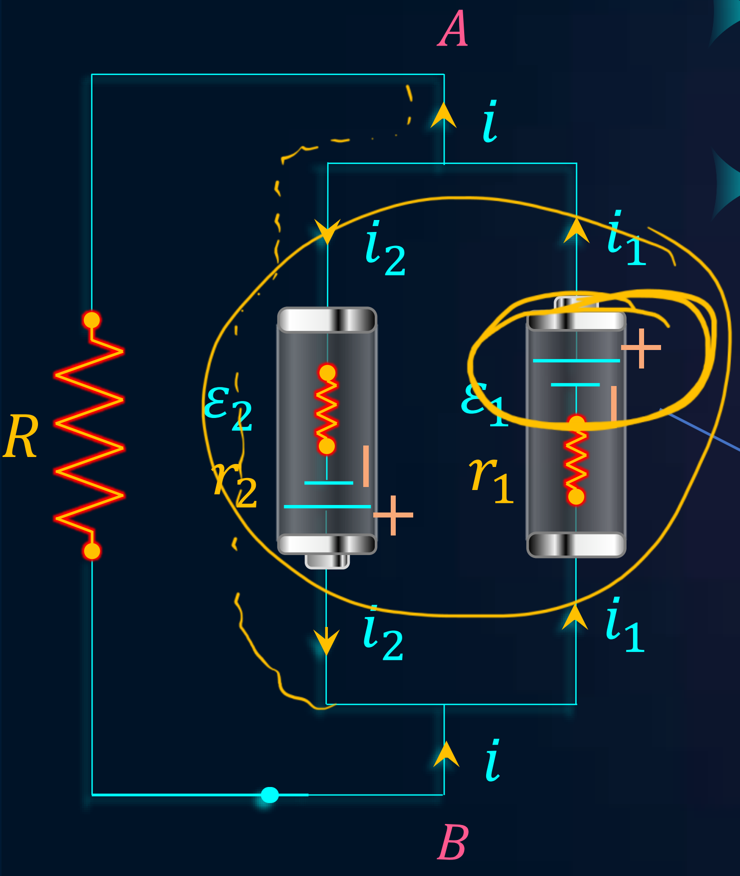
Parallel Combination of Cells

$$i = i_1 - i_2$$

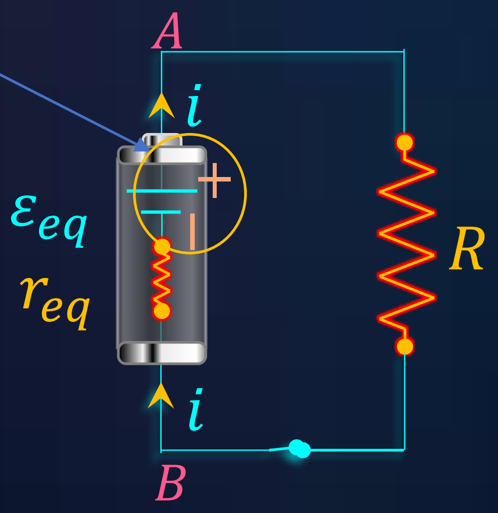
- Opposite polarity

- Suppose $\varepsilon_1 > \varepsilon_2$

$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} - \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = V$$



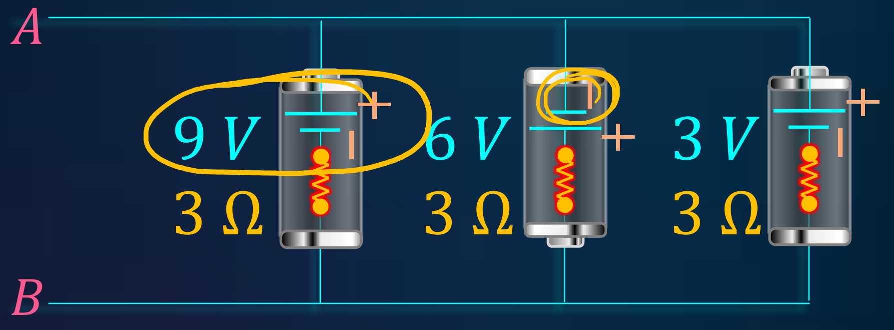
Note: — Direction of current depends on ε_1 and ε_2.



$$\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}}$$

$$\boxed{\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} - \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}}$$

Parallel Combination of Cells



$$r_{eq} = 1\,\Omega$$

$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} - \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$3 - 2 + 1 = 2\,V$$